Seismic Simulations

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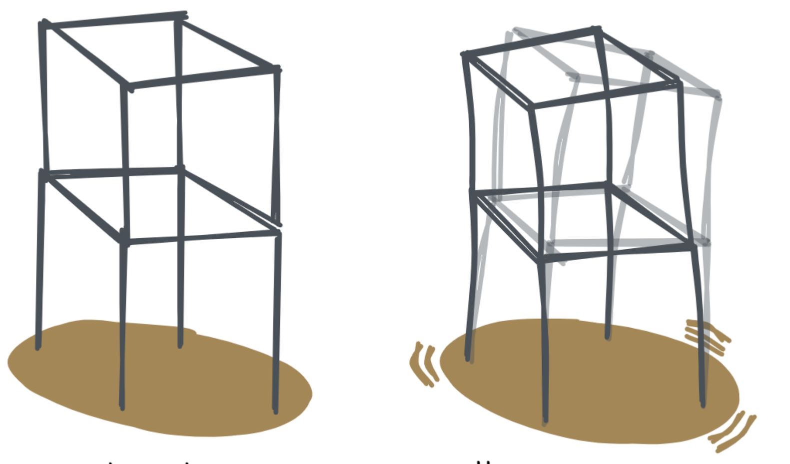
Implementing and Improving a new Method for Dynamic Structural Analysis

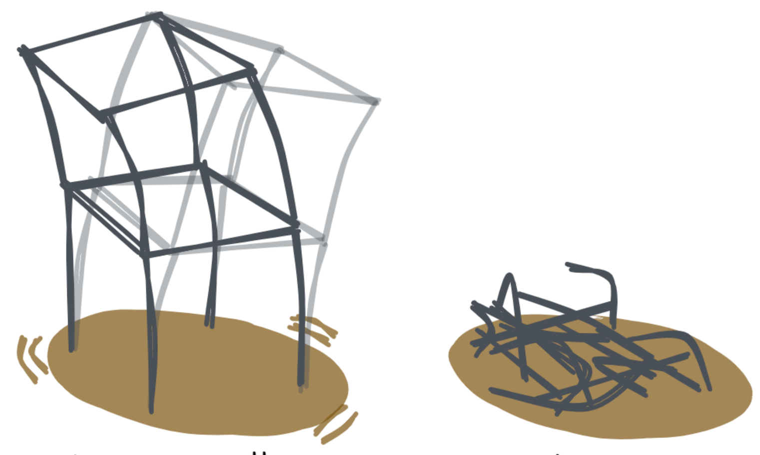
Structures and Vibrations



A typical building frame made of steel beams... ...will begin to shake under excitation...

All structures vibrate. If you imagine striking a tuning fork, it vibrates at a particular frequency. Buildings are the same. All structures have some natural frequencies.

If a building gets some external force acting at one of its natural frequencies, the building will begin to resonate. This means the building vibrates more and more until it shakes itself apart!



..and potentially begin to resonate... ...and this can lead to collapse!

Structural Modelling at Scale

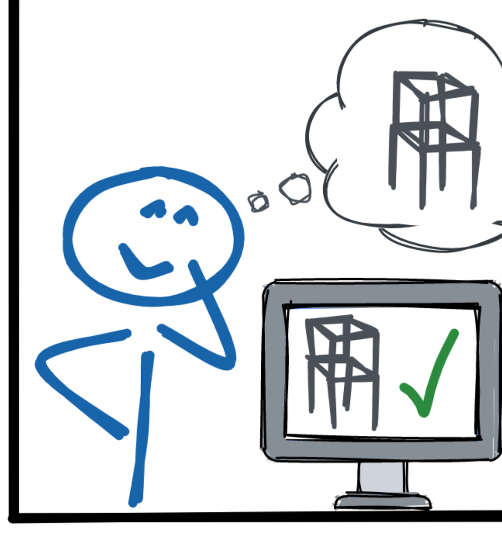
Structures need to be modelled and simulated to predict their behaviour.

These simulations use Finite Element Methods (FEM).

Can the building support its own weight?

How do vibrations move through the structure?

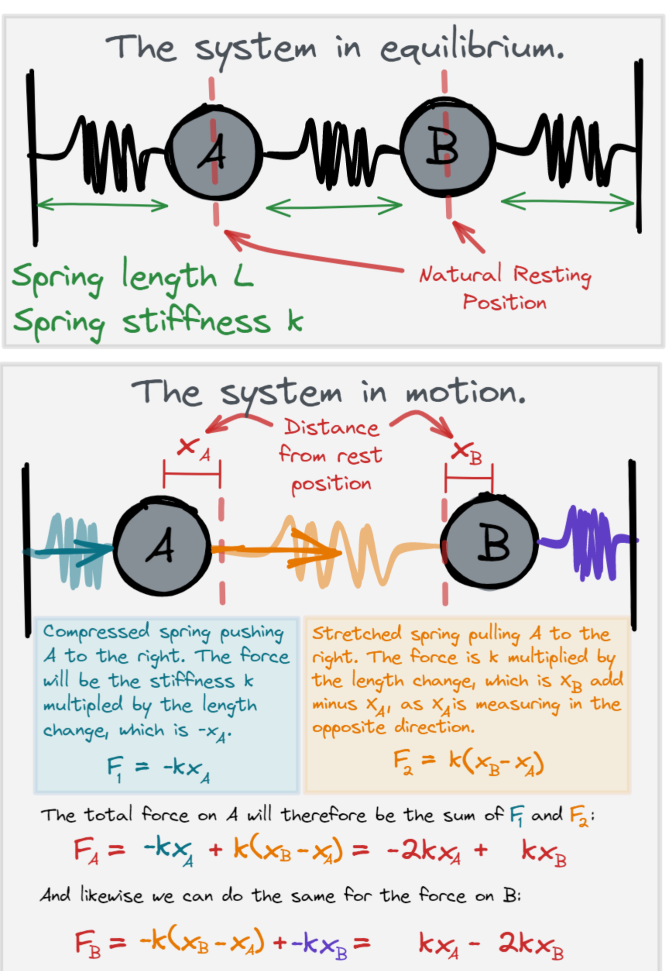
Arup develops FEM software for engineers to use to model and simulate designs.



At the core of these tests are complex mathematical solvers

Our aim is to try and create solvers with increased accuracy and efficiency.

Modelling Vibrations and Acceleration



We have the mathematical tools that let us model how structures move.

Hooke's Law

$$F = kx$$

Force from the spring · Stiffness · Change in spring length

Lets us calculate the force from the spring

Newton's 2nd Law

$$F = ma$$

Force on object · Mass of object · Acceleration of object

Lets us figure out what the force does

Then we can divide the force from the springs by the mass to find out the acceleration of the objects. From this we can predict how it'll move.

Matrices and Eigenvalues

We can rewrite systems of equations as structures called Matrices.

$$\begin{aligned}-2kx_A + kx_B &= F_A\\ kx_A - 2kx_B &= F_B\end{aligned} \rightarrow \begin{pmatrix}-2k & k\\ k & -2k\end{pmatrix}\begin{pmatrix}x_A\\ x_B\end{pmatrix} = \begin{pmatrix}F_A\\ F_B\end{pmatrix} \rightarrow \frac{k}{m}\begin{pmatrix}-2 & 1\\ 1 & -2\end{pmatrix}\begin{pmatrix}x_A\\ x_B\end{pmatrix} = \begin{pmatrix}a_A\\ a_B\end{pmatrix}$$

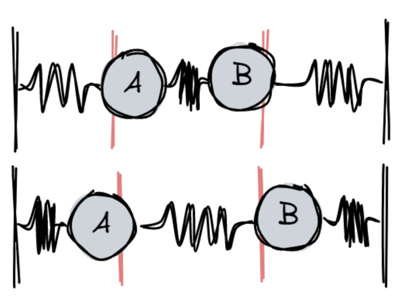
Displacement · Force · Acceleration

Resonance is when the displacement is proportional to the acceleration.

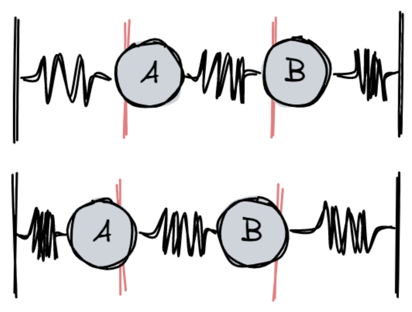
$$\frac{k}{m}\begin{pmatrix}-2 & 1\\ 1 & -2\end{pmatrix}\begin{pmatrix}x_A\\ x_B\end{pmatrix} = \lambda\begin{pmatrix}x_A\\ x_B\end{pmatrix}$$

Solutions to this equation are called the eigenpairs of the matrix.

$$\lambda = -3\frac{k}{m} \qquad \begin{pmatrix}x_A\\ x_B\end{pmatrix} = \begin{pmatrix}x\\ -x\end{pmatrix}$$



$$\lambda = -\frac{k}{m} \qquad \begin{pmatrix}x_A\\ x_B\end{pmatrix} = \begin{pmatrix}x\\ x\end{pmatrix}$$

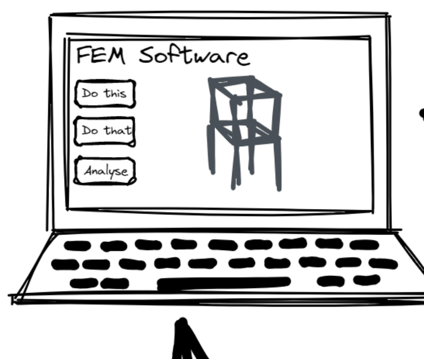


Eigensolvers for Structural Analysis

Real structures have millions of variables.

So we have huge matrices.

These problems are often too large to solve exactly.



Sparse Symmetric Generalised Eigenvalue Problem

Find the useful eigenpairs of:

$$\begin{pmatrix}k_{11} & k_{12} & \cdots & k_{1N}\\ k_{21} & k_{22} & & \vdots\\ \vdots & & \ddots & \\ k_{N1} & \cdots & & k_{NN}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ \vdots\\ x_N\end{pmatrix} = \lambda\begin{pmatrix}m_{11} & m_{12} & \cdots & m_{1N}\\ m_{21} & m_{22} & & \vdots\\ \vdots & & \ddots & \\ m_{N1} & \cdots & & m_{NN}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ \vdots\\ x_N\end{pmatrix}$$

eigenvalue · eigenvector

Eigensolver

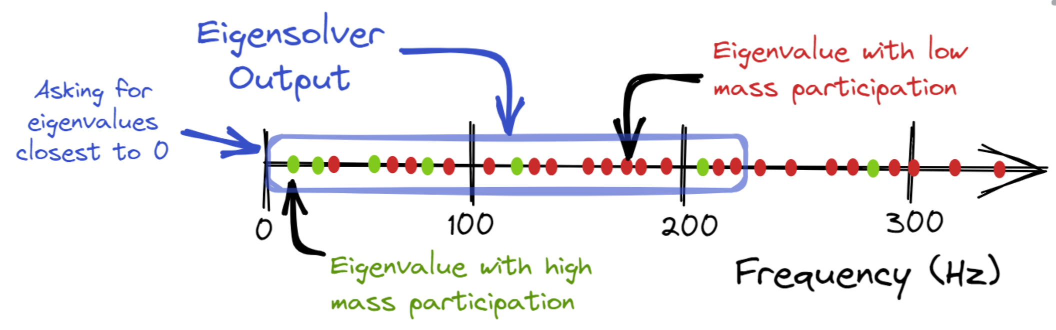
Instead, we use iterative solvers. These go through the problem and produce estimates for the eigenpairs. They repeat over and over again, improving the estimates with each pass, until they're sufficiently accurate, when we say they're converged.

Using Iterative Eigensolvers

Most iterative eigensolvers find the eigenpairs closest a given value. However the engineers don't necessary want this. They are looking for the eigenpairs with the highest mass participation.

This is a measure of how the mass of the building moves when it vibrates. This is critical to analysis of earthquakes.

National Standards tell us we need to analyse the building in such a way that we capture 90% of the total mass participation.



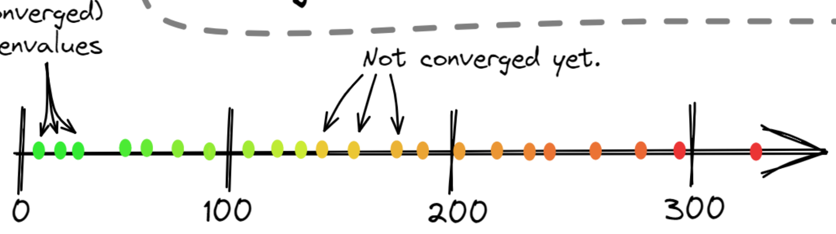
Here we can get 90% mass participation with the solver output, but we would also get it with just the green frequencies.

Current eigensolvers waste resources on the unimportant eigenpairs.

If we can avoid this, the solver will run faster, use less memory and give a more meaningful result to the engineer!

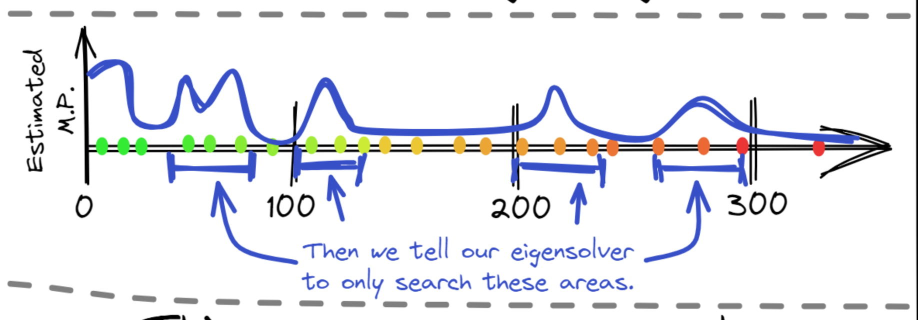
Our New Solver: MASIL

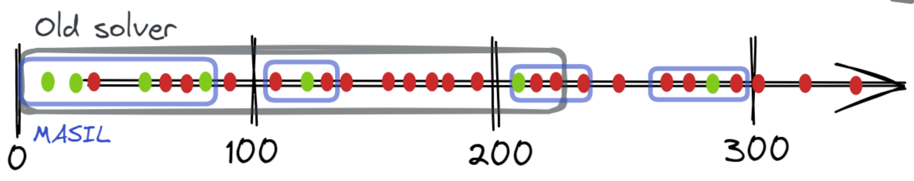
We have developed a new pre-processing step for an eigensolver that gives us an idea where the useful eigenpairs are.



After several iterations, some eigenpairs have converged, and the rest are getting there.

We discovered a way to estimate mass participation accurately with unconverged eigenvalues.





This means we can search over much smaller areas until we find the necessary eigenvalues.

All this makes for a solver that is quicker, more efficient and more accurate!

From Idea to Implementation

2015

Arup and the University of Manchester discuss a new algorithm that will find eigenpairs with high mass participation.

2015-2019

PhD funded at the University of Manchester to study this problem, MASIL is discovered.

2020-2022

Knowledge Tranfer Partnership undertaken with Arup, the University of Manchester and InnovateUK to implement the MASIL algorithm in Arup's FEM software. Further improvements to MASIL are developed and it is successfully tested and deployed into commercial software to be used by engineers.

2023+

Arup and the Unversity of Manchester continue to work together to implement and improve algorithms for other modelling problems.